NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1434

EXTREME SPEEDS AND THERMODYNAMIC STATES IN SUPERSONIC FLIGHT

By Klaus Oswatitsch

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SUMMARY

The increasing importance of high-speed flow leads to similar problems in various fields of research which are summarized in what follows. Typical of all cases is the conversion of high kinetic energy into extreme thermodynamic states with temperatures of several thousand degrees, frequently connected with dissociation and ionization of the gas involved. There is also a characteristic small sensitivity to the processes discussed in the case of gases of low molecular weight (light gases).

The penetration of meteors into the atmosphere of the earth at astronomical speeds results in temperatures higher than those of the surface of the sun. Such temperatures may be produced in shock tubes, with light gases used as the driving gas. For supersonic fighters the problem of propulsion is less difficult to solve than the problem of large heating, on the surface and in the combustion chamber. Finally, for the space-travel rocket, astronomical speeds have to be reached which require the lightest possible gases as propellants. Here again, dissociation processes in the combustion chamber are of considerable importance.

1. INTRODUCTION

If science deals with extreme conditions, it may appear, at first glance, as though the record-crazy present is penetrating into the domain of serious research. On the one hand, this is actually the case. Research cannot and should not keep aloof from the problems of extreme technical questions, and the following expositions are meant to be a critical evaluation of publicly discussed related plans. On the other hand, the study of

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extreme conditions holds a special lure for research because it leads to new and unexpected effects. Regions are reached which, at first, challenge engineering and physical intuition born of experience. Obstacles appear in unexpected places while expected difficulties often vanish. The arrogance which makes us push on into hitherto untouched fields turns into humility in the face of the unknown and the unexplored.

In the following discussions we attempt a maximum of clarity with a minimum of mathematical and physical complexity. The results given can be found in various reports in various issues of the international scientific literature. Only a few papers are quoted explicitly. A specialist in this field will gain in what follows - aside from a new compilation - at most a few new approximation formulas. However, the author hopes to find his most appreciative readers among the experts in neighboring fields and among interested novices. Exact derivations have frequently been added in small print.

2. ASTRONOMICAL SPEEDS AND THERMODYNAMIC STATES

Table 1 shows, in addition to the velocity of sound c and the velocity of propagation of light, approximate values for astronomical speeds. In order to have a relationship to the customary velocity scale of modern engineering, the velocity of sound is given also in kilometers per hour (km/h).

Compared to technologically realized speeds, we deal in the universe actually with "astronomical" numbers. Modern pursuit planes attain sonic velocity and will be considerably faster within the next few years. Rifle projectiles, antitank and antiaircraft shells as well as naval-gun projectiles have initial speeds of about three times the velocity of sound, and modern liquid-fuel rockets reach seven times, perhaps even ten times sonic velocity. However, when the earth, on its path around the sun, encounters a piece of matter, this latter enters the earth's atmosphere with a velocity of 30 km/sec or about 100 times the speed of sound. A shooting star or a meteor results. The air is not capable of avoiding it in time, since a small pressure disturbance is propagated only with sonic velocity which is far exceeded by the meteor flying at high supersonic speed. The meteor pushes the air in front of it in a cushion of high density. Ahead of the meteor, density, temperature, and pressure jump, shocklike, in a bow wave (fig. 1) to a multiple of their initial values. We speak of a compression shock or, abbreviately, of a shock.

Here the importance of the ratio of the velocity W and the velocity of sound c becomes clear. This ratio is generally designated as Mach number M, after the Viennese physicist and philosopher Ernst Mach

$$M = \frac{W}{c} \tag{1}$$

E. Mach was one of the first to recognize the significance of this proportionality factor for flow problems.

The pressure increase in the perpendicular part of the bow wave can be easily determined approximately. We visualize ourselves placed on the meteor as observers. The pressure increase in the bow wave originates by the air, flowing against the meteor with enormous power, being suddenly brought to an almost complete stop. The momentum per unit volume is ρW (ρ = density). A momentum stream $\rho W \times W$ flows through a unit area per unit time. The rate of loss of this momentum is equal to the static pressure $\hat{p}$ immediately behind the front of the bow wave

$$\rho W^2 = \hat{p} \tag{2}$$

It is true that static pressure exists also ahead of the bow wave, and a small momentum stream is present also behind the bow wave since the air must flow off around the body. However, both quantities named are much smaller than the effects included in (2).

For ideal gases, there exist the following speed-of-sound formulas: (T = absolute temperature, $\kappa = c_p/c_v$ ratio of the specific heats for constant pressure c_p and constant volume c_v, m = molar weight, R = universal gas constant)

$$c^2 = \frac{\kappa p}{\rho} = \frac{\kappa RT}{m} = \kappa\left(c_p - c_v\right)T \tag{3}$$

Ahead of the shock, the air is certainly to be regarded as an ideal gas. Hence there follows from equation (2) with equations (1) and (3) the following formula for the pressure rise in the bow wave for high Mach number

$$\frac{\hat{p}}{p} = \frac{\rho W^2}{p} = \kappa M^2 \tag{4}$$

Formula (4) shows clearly the importance of the Mach number; even though it is valid only for higher supersonic speeds - the so-called hypersonic speeds - it has the advantage of not containing an assumption on the gas state after the compression. This is important, for in the case of the meteor flying at Mach number 100 there results (κ is the value for air ahead of the bow wave: $\kappa = 1.4$) a pressure increase of

$$\frac{\hat{p}}{p} = 14{,}000$$

thus an extremely high value.

An exact calculation leads to the following formula: Since an equal mass flows in ahead of the shock front as flows off behind the shock front (see, for instance (ref. 1), p. 20), the continuity condition

$$\hat{\rho}\hat{W} = \rho W \tag{5}$$

applies. The exact momentum equation reads

$$\hat{p} + \hat{\rho}\hat{W}^2 = p + \rho W^2 \tag{6}$$

From equations (5) and (6) follows

$$\frac{\hat{p}}{p} = \frac{\rho W^2}{p}\left(1 - \frac{\hat{W}}{W}\right) + 1 = \kappa M^2\left(1 - \frac{\rho}{\hat{\rho}}\right) + 1 \tag{7}$$

Whereas the last term has no significance whatsoever for high speeds, the parenthesis in equation (7) in the given example leads, according to table 2, to a correction of about 10 percent compared to equation (4).

In order to form now also an opinion about the temperature increase occurring in the bow wave, we shall go back - as in all temperature problems - to the energy theorem. Let us consider the unit mass of a gas particle which passes through the front of the bow wave (fig. 2). The kinetic energy $W^2/2$, and also the internal energy of the unit mass e are released; furthermore, the gas flowing out at the pressure p performs work of the magnitude p/ρ by displacement of the volume $1/\rho$ per unit mass. On the other side of the bow-wave front corresponding energy increases have to be considered; hence there results the energy balance

$$\hat{e} + \frac{\hat{p}}{\hat{\rho}} + \frac{\hat{W}^2}{2} = e + \frac{p}{\rho} + \frac{W^2}{2} \tag{8}$$

This is valid under the assumption made here and always made later, that no energy radiates toward the sides; however, at very high temperatures, this is probably only conditionally true and will probably require a correction. In equation (8), the kinetic energy behind the bow wave $\hat{W}^2/2$ is, in practice, quite insignificant. If the energy consideration is set up, instead of for the state behind the bow-wave front, for the state at the stagnation point, there even applies exactly (with the subscript 0 for the state of rest $W = 0$: $e = e_0$, $p = p_0$, $\rho = \rho_0$)

$$e_0 + \frac{p_0}{\rho_0} = e + \frac{p}{\rho} + \frac{W^2}{2} \tag{9}$$

For an ideal gas there applies, furthermore, the equation of state

$$\frac{p}{\rho} = \frac{RT}{m} = \left(c_p - c_v\right)T \; {}^{*} \tag{10}$$

and it will be shown that, in what follows, the gases may very well be regarded as "ideal."

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*NACA reviewer's note: Although the first equality of equation (10), $\frac{p}{\rho} = \frac{RT}{m}$, is applicable in the general case of varying molecular weight, the second equality, $\frac{RT}{m} = \left(c_p - c_v\right)T$, is not.

Furthermore, if we assume, in order to arrive at numerical values, that we are dealing with an ideal gas of constant specific heat, there applies for the internal energy, aside from an additive constant, $e = c_v T$. Hence it follows, together with equation (10), that

$$c_p T_0 = c_p T + \frac{W^2}{2}$$

or, introducing the velocity of sound with equation (3) and the Mach number with equation (1)

$$\frac{T_0}{T} = 1 + \frac{\kappa - 1}{2} M^2 \tag{11}$$

Hence, for air ($\kappa = 1.4$) at a meteor Mach number $M = 100$, it follows that the temperature ratio $T_0/T = 2,000$; for an absolute temperature of 225° as prevails, for instance, in the stratosphere, this would correspond to a rest or stagnation-point temperature of $T_0 = 450,000°$. At such a temperature, however, the air has certainly been dissociated or ionized long before. The assumption of the air as being a gas of constant specific heat, which forms the basis of equation (11), has certainly been grossly violated. This is equally true for the state immediately behind the shock front where the air also has come almost to a standstill and where, therefore, almost stagnation conditions exist.

Whereas the momentum considerations with equation (4) lead, therefore, to a quite serviceable estimate for the pressure increase in the bow wave, a more exact knowledge of the gas state is necessary in order to arrive at results for the temperature. Figure 3 shows the composition of the air as a function of the temperature for normal density according to quantum-theory calculations of Burkhardt (ref. 2).* According to these calculations, the composition of the air of the normal state of approximately 20 percent O_2 and 80 percent N_2 is maintained up to beyond $T = 2,000°$ absolute. Aside from a slight formation of NO, N_2 and O_2 start dissociating to N and O at about 3,000°, and at 10,000° the air is a monatomic mixture of nitrogen and oxygen. The mean molar weight m has varied inversely as the number of molecules and has at 10,000°, therefore, only half the magnitude of the normal state. In cases of heating beyond 10,000°, the gas is ionized. Due to the formation of electrons, the molar weight decreases further. Figure 4 shows the internal energy of $m = 29g$ air. This is, therefore, the internal energy of a mole at low temperatures and, according to the kinetic gas theory for molecules with five degrees of freedom, thus for air of low temperatures,

$$me = 5T$$

is valid.

*NACA reviewer's note: Since the calculations of Burkhardt are now outmoded, figures 3 and 4 are only approximately correct.

Since the molar weight decreases at higher temperatures, me is, at higher temperatures, no longer the internal energy of a mole. Figure 4 shows that the internal energy increases considerably more than for the "ideal gas of constant molar weight." This is caused by the large energies required to first split the molecules into atoms and then to separate electrons from the atoms; such energies are comparable to those of intensive chemical reactions.

In the entire range considered, the air is to be regarded as an "ideal gas" in the sense that the equation of state (10) applies with substitution of the particular molar weight m, which is a function of the temperature and also of the density. From the standpoint of gas kinetics, this means that the volume of the molecules is always small compared to the volume of the space in between and that the forces of mutual attraction are insignificant. The dependence of the internal energy on the density is only a result of the dependence of the decomposition of the gas on the density. The potential energy arising from the mutual attraction of the molecules, in contrast, does not play any role, just as in the case of the ideal gas.

For calculating the change of state in the shock wave, we shall use a relationship between the thermodynamic state variables which is known as "dynamic adiabatic." W and $\hat{W}$ can be eliminated from the three equations (5), (6), and (8) which are valid for all media, and

$$\hat{e} - e = \frac{1}{2}\left(\frac{1}{\rho} - \frac{1}{\hat{\rho}}\right)(\hat{p} + p) \tag{12}$$

results. For the extreme states here treated, $\hat{p} \gg p$ and, according to figure 4, likewise $\hat{e} \gg e$, whence we obtain the following equation, of good approximation:

$$\frac{\hat{\rho}}{\rho} - 1 = \frac{2\hat{e}\hat{\rho}}{\hat{p}}$$

and, with use of the equation of state of ideal gases (10) the approximation

$$\frac{\hat{\rho}}{\rho} - 1 = \frac{2\hat{e}\hat{m}}{R\hat{T}} = \frac{2\hat{e}m}{R\hat{T}}\,\frac{\hat{m}}{m} \tag{13}$$

The difficulty lies in the fact that the state after the compression is desired but that there the internal energy $\hat{e}$ is dependent on $\hat{T}$ and $\hat{\rho}$ in a complicated manner. Therefore, we shall simply assume the state $\hat{\rho}$ and $\hat{T}$ and calculate the Mach number of the meteor as well as its flight altitude, thus the air density ρ in which it is flying.

The second column of table 2 gives the assumed temperature $\hat{T}$ multiplied by the absolute gas constant. We put there $R = 2$ cal/degree, not 2 cal/g × degree, because the molar weight has been introduced with the dimension g (gram) in figure 4. Columns 3 and 4 follow from figures 4 and 3, column 5 from formula (13). It is remarkable here that the density in the bow wave increases only by a factor of 6 to 13. This is of course not surprising since it is known that the density in the compression shock of an ideal gas of constant specific heat of $\kappa = 1.4$, in the limiting case of high Mach number, increases only to 6 times the initial value. The dissociation and ionization of the gas thus causes an increased compression and a decreased rise in temperature - as will be shown later - and has no important influence on the pressure rise.

As a result of the slight compression of the gas, an ideal gas may still be regarded as "ideal" even after the compression shock - however, with variable molar weight m, dependent on the state.

Since $\hat{\rho}$ was assumed as the normal density, column 5 also determines the density ahead of the bow wave and thus the flight altitude of the body. It is represented with the assumption of the so-called "standard atmosphere" (ref. 3). In every case, we are dealing with flight altitudes in the stratosphere, thus with altitudes which are of interest not only for meteors but also for missiles flying at very high speeds. There a temperature of approximately $T = 223^{\circ}$ absolute prevails. Thereby the temperature ratio also is fixed. With the equation of state of ideal gases (10) ahead of and behind the front of the bow wave there then follows the pressure ratio

$$\frac{\hat{p}}{p} = \frac{\hat{T}}{T} \frac{\hat{\rho}}{\rho} \frac{m}{\hat{m}} \tag{14}$$

and with equation (7) (or with the less accurate equation (4)) the Mach number. Because of lack of the m-values for $T = 30{,}000^{\circ}$, the calculation in table 2 is limited to Mach numbers $M \leqq 44$. On the other hand, a meteor in the upper stratosphere is strongly decelerated so that table 2 is of interest for this astronomical phenomenon, too. We shall encounter the Mach-number range between 15 and 30 below also in the problem of satellites. For this reason, the results are of technical interest as well. Of course, they are of importance only for the pressure region behind the bow wave. Considerably weaker effects would occur on projectiles with conical points.

As mentioned before, the pressure rise depends in practice only on the Mach number. From $M = 30$ (line before last in table 2) there follows with equation (4) $\hat{p}/p = 1{,}260$. In equation (14) the left side is thus fixed with M. The compression is approximately twice as high as for an ideal gas with $\kappa = 1.40$ and the molar weight only half that in the initial state. Hence the temperature rise in this case is only about

1/4 of that calculated with equation (11); thus, instead of $\hat{T}/T \approx T_0/T = 180$, only 45. For an approach-flow temperature of $T = 223^\circ$, this means, instead of a temperature of $T = 40,000^\circ$, only $10,000^\circ$ behind the bow wave - a value which, of course, is still far above the temperature of the surface of the sun. The air is practically dissociated completely in this case.

It is true that we make here the assumption that the time intervals during which the gas stays in the state considered are sufficient for adjustment of the thermodynamic equilibrium, the dissociation and ionization equilibrium on which figures 3 and 4 are based. Experiments with the shock tube treated in section 4 have shown that relaxation phenomena do appear. The final state is therefore attained, sometimes, not immediately behind the shock front but only after passing through a transitional zone. It must be taken into consideration, though, that a delay in the disintegration of the molecules causes a superelevation of the temperature. The tendency of the gas toward dissociation and ionization is therefore greatly heightened in this transitional state which makes the gas there tend toward thermodynamic equilibrium the more rapidly.

3. DRAG PROBLEM

The drag of bodies flying at such high speeds is governed exclusively by the extraordinarily high pressures in the neighborhood of the stagnation point. These pressures amount to a multiple of the approach-flow pressure. In the face of these facts, even a complete vacuum behind the maximum thickness of the body cannot produce a noticeable suction. Friction forces can exert a certain amount of influence only in the case of very slender bodies which will not be considered here.

Since the pressure at the stagnation point lies above the pressure $\hat{p}$ behind the perpendicular part of the bow wave (the pressure at the maximum body thickness, however, is considerably smaller than $\hat{p}$) the drag D will be equal to the cross-sectional area F times a mean pressure $\bar{p} = C\hat{p}$ where C is of the order of magnitude of unity. Hence, there follows with equation (2)

$$D = F\bar{p} = CF\hat{p} = CF\rho W^2 \qquad (15)$$

Thus the drag coefficient c_D referred to the "frontal area" F is

$$c_D = \frac{D}{F\rho W^2/2} = 2C \qquad (16)$$

This result is noteworthy. It indicates that, in the range of very high Mach numbers, the drag coefficient becomes independent of M. The author generally proved this result (ref. 4) for ideal gases of constant specific heat where it was shown that, for blunt bodies, the independence of M appears for $M^2 \gg 1$. The tests (ref. 5) show indeed that the coefficients for a sphere and for a cylinder in an axial approach flow, as early as for $M = 3$, approach an asymptotic final value which lies at $c_D = 0.95$ for the sphere and at $c_D = 1.65$ for the cylinder (fig. 5). The corresponding factors $C = \hat{p}/\bar{p}$ are $C = 0.48$ and $C = 0.82$. Another conclusion is surely permissible here. Since the pressure increase in the perpendicular part of the shock depends only on the Mach number M and is only insignificantly influenced by the dissociation of the gas, the effect of dissociation and ionization on the drag coefficient of the gas certainly is also slight. This must apply all the more, the slenderer the body, because then the gas also deviates less from the state of constant specific heat. The influence of κ on c_D, too, can only be slight, according to equation (16).

At this point we want to discuss briefly the problem of drag of a meteor where the pure kinetic energy is compared, on the one hand, to the work done against drag in penetrating the atmosphere, and on the other hand, to the heat of fusion of iron, the chief constituent of most meteors. Referred to the unit mass, the kinetic energy is simply $W^2/2$ (table 3). With equation (16), the resistance work in a distance L, at the density ρ, for a sphere of the radius r (a meteor is to be regarded, approximately, as such a sphere), is

$$DL = c_D r^2 \pi \rho L \frac{W^2}{2} = 0.95 r^2 \pi \rho L \frac{W^2}{2}$$

The resistance work for a unit mass then is, with ρ_E as the density of iron

$$\frac{DL}{(4/3) r^3 \pi \rho_E} = 0.71 \frac{1}{r} \frac{\rho L}{\rho_E} \frac{W^2}{2}$$

The height of the "constant-density atmosphere" ($\rho = 1.293 \times 10^{-3}$ g/cm^3 is 8 km. Since the product ρL is not dependent on the assumption of ρ (the weight loading per unit area of the ground is the same in any case) we may assume the estimate with a mean constant density to be quite usable. With $\rho_E = 7.8$ g/cm^3 there then follows the second value of table 3. The radius of the meteor r, expressed in cm, is to be substituted into the formula.

Third in the table, finally, we find the heat of fusion of iron. As a quantity having the dimension cal/g, it may just as well be given in the form of a kinetic energy per unit mass. Here and later on, heats of transformation are thus expressed mostly in the dimensions $\left[1/2(\text{km/sec})^2\right]$, that is, they correspond to the kinetic energy per unit mass of a body when its velocity is counted in km/sec. Thus

$$\left\{\begin{aligned}\left[\text{cal/g}\right] &= 0.4186 \times 10^{8}\left[(\text{cm/sec})^{2}\right]\\ &= 0.4186 \times 10^{-2}\left[(\text{km/sec})^{2}\right]\\ &\doteq 0.09150^{2}\left[\tfrac{1}{2}(\text{km/sec})^{2}\right]\end{aligned}\right\} \tag{17}$$

1 cal/g corresponds, therefore, to the kinetic energy per unit mass of a body flying with a velocity of 0.09150 km/sec. This at first unusual manner of expression will prove advantageous for energy considerations in the field of rockets.

For a meteor of a radius of $r = 94$ cm or a diameter of 2m, the kinetic energy and the resistance work are about the same, that is, the boundary where the meteor is brought to a complete stop by the earth's atmosphere lies at this diameter. It is true that the pull of gravity has not been taken into consideration here, but in the case of a meteor it is of little importance. The kinetic energy of a meteor flying in at 30 km/sec is approximately 2,000 times ($30^2/0.41 = 2{,}200$) its heat of fusion. A spherical body measuring many meters will thus penetrate the earth's atmosphere but will not only melt but also evaporate at the impact. A small spherical body generates on its way through the atmosphere - due to its resistance work - so much heat that even a small part of this heat is sufficient to melt it. It is therefore not surprising that at least the point of maximum temperature rise - the head of the meteor - must melt.

The considerations on resistance work are associated especially with conditions for meteors; thus they do not permit immediate conclusions as to the processes in the case of projectiles or rockets. For these latter, the drag values per unit mass are sometimes considerably smaller.

4. SHOCK TUBES

The considerations following next will be devoted to the problem of producing extreme states in the laboratory. For the various customary types of supersonic wind tunnels, one encounters many difficulties in the attempt to achieve hypersonic velocities. The problem of the energy required is not so difficult. Considerable pressure gradients also can be achieved; however, the appearance of very high temperatures poses technical problems which are very hard to solve. In recent years, a test arrangement which, in English, is called "shock tube" has attained importance. We deal here with the production of a one-dimensional explosion in the laboratory so that the instrument should perhaps best be called an "explosion tube" in German.

First, let us consider the unsteady propagation of a two-dimensional sound wave (fig. 6). If we deal not with a wave-shaped but with a step-shaped disturbance, the sound wave produces a small disturbance of the thermodynamic state and of the velocity of sound c and causes a change in the flow velocity W. In the reference system of the sound wave, the connection between temperature variation and variation of kinetic energy was previously indicated in the derivation of equation (11). This connection applies for any stationary disturbances, compressions or expansions

$$W\,dW = c_p\,dT^{*}$$

or, with introduction of the velocity of sound with equation (3)

$$W\,dW = -\frac{2}{\kappa - 1}\,c\,dc \tag{18}$$

However, we now deal with the disturbance in a sound wave. In order to keep the latter stationary in figure 6, the flow velocity must be chosen equal to the velocity of sound. Hence, for the change of state in a sound wave there applies in the coefficients of equation (18): $W = c$. The change of state in a sound wave of an ideal gas of constant specific heat is given exactly by

$$dW = -\frac{2}{\kappa - 1}\,dc \tag{19}$$

Thus, the velocity varies in a sound wave in the case of air ($\kappa = 1.40$) five times as much as the velocity of sound. For equation (19) it no longer matters whether the sound wave is considered in a coordinate system fixed to the wave step or in an arbitrarily moved coordinate system, whether the wave step travels on or remains stationary; a velocity difference dW is independent of the selection of the coordinate system. A wave crest, for instance like the one sketched in figure 7, with a state of rest ($W = 0$, $c = c_0$) on the left side, may be regarded as a superposition of sound waves, each of which travels in the state created by the preceding wave. In figure 7, for instance, the first wave, situated farthest to the left, runs with the velocity of sound of the state of rest c_0. The decrease of the velocity of sound in the first "partial wave" - which is only an expression of the drop in pressure and density - leads, according to equation (19), to the gas beginning to flow slightly to the right. On this gas flowing to the right, the next sound-wave step now moves to the left, etc. Thereby we also obtain immediately a picture of the deformation with time of the sound wave. The absolute velocity of a sound wave moving to the left is $W - c$. For

*NACA reviewer's note: This equation should read $W\,dW = -c_p\,dT$.

a state of rest, $W = 0$, it runs with the velocity $-c$; for $W = c$, it remains stationary. By integration of equation (19), the flow velocity of the gas is readily expressed by c. Taking $c = c_0$ for $W = 0$ there applies

$$W = \frac{2}{\kappa - 1}\left(c_0 - c\right) \tag{20}$$

Whereas thus the velocity of sound in the wave of finite amplitude decreases, the flow velocity increases. With equation (20), there applies for the absolute velocity of the sound wave

$$W - c = \frac{2}{\kappa - 1}\left(c_0 - c\right) - c = -c_0 + \frac{\kappa + 1}{\kappa - 1}\left(c_0 - c\right) \tag{21}$$

For very strong expansion, that is, for very low temperature and a very small value of c, the part of the wave situated on the right is carried off to the right by the gas flow; that is, the wave flattens with time.

In the consideration of an "explosion" in a tube, this appears self-evident. Let us start from an initial state in which on the left side of a membrane set into a tube (fig. 8) high pressure exists while the other side is pumped out. Shortly after the membrane is burst, a pressure distribution similar to the one sketched and a corresponding sonic-velocity distribution exist. The further variation with time is that which was discussed with the aid of figure 7. Whereas the pressure on the left side is reduced by the progressing expansion wave, the high-pressure gas flows, explosively, into the low-pressure side. The attainable maximum velocity corresponds to a complete expansion $p \rightarrow 0$, $\rho \rightarrow 0$, $T \rightarrow 0$, $c \rightarrow 0$; therefore, according to equation (20)

$$W_{max} = \frac{2}{\kappa - 1} c_0 \tag{22}$$

It is true that a pressure gradient sufficient for attaining this maximum value is hard to achieve so that we must limit ourselves to considerably smaller values. The gas expands isentropically (adiabatically). For this there applies, at $\kappa = \text{const}$

$$\frac{c}{c_0} = \left(\frac{T}{T_0}\right)^{1/2} = \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{2\kappa}} \tag{23}$$

or for the velocity to be attained for a pressure ratio p/p_0 with equation (20)

$$\frac{W}{c_0} = \frac{2}{\kappa - 1}\left[1 - \frac{c}{c_0}\right] = \frac{2}{\kappa - 1}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{2\kappa}}\right] \tag{24}$$

According to equation (24), a high velocity of sound at rest is no less important for a high flow velocity W than a large pressure drop. This rest velocity cannot be too greatly increased by heating since a doubling of c requires, according to equation (3), a quadrupling of the absolute temperature. However, precisely this equation (3) shows that gases of smaller molar weight have a considerably higher c. For equal temperature, hydrogen $\left(H_2\text{:}\ \kappa = 1.40,\ m = 2\right)$ has $\sqrt{29/2} = 3.8$ times the sonic velocity of air (air: $\kappa = 1.40$, $m = 29$); this phenomenon is explained, from the standpoint of gas kinetics, by the considerably higher flight velocity of the molecules of smaller mass.

If the high-pressure side of the shock tube is filled with H_2 and the low-pressure side with very highly rarefied air, the hydrogen rushes, after bursting of the membrane, with great speed into the low-pressure side, displacing the air which is compressed in a shock. Due to the large pressure differences, the process can be sketched only with a distorted pressure scale in figure 9. The state of rest of the air is designated by the subscript 1 in order to distinguish it from the state of rest of the hydrogen. At the surface of contact of the two gases, pressure and velocity must be identical. For H_2, the connection between W and p is given by equation (24). For a moving shock, however, it still has to be derived. So as not to lose any clarity, the consideration will be limited to the case of strong shocks treated here. For the stationary flow there applies equation (2), with the flow velocity behind the shock $\hat{W}$ being rather small, compared to W.

The shock front need only be considered from a coordinate system fixed to the approaching gas in order to arrive at the case of a shock moving into a gas at rest, as in figure 9. The shock front then has the velocity W, and - in the case of a strong shock - the gas behind the shock front also has approximately the same velocity. Since the values ahead of the shock are written with the subscript 1, and those behind the shock without subscript, there applies therefore for the moving strong shock

$$W^2 = \frac{p}{\rho_1} = \frac{1}{\kappa_1}\kappa_1\frac{p_1}{\rho_1}\frac{p}{p_1} = c_1^{\,2}\frac{1}{\kappa_1}\frac{p}{p_1} \tag{25}$$

For judging the merit of equation (25), we shall briefly derive the exact formula. It is easy to derive, from the exact equations (5) and (6), the equation (26) for stationary flow

$$W = \sqrt{\frac{\hat{\rho}}{\rho}\frac{\hat{p} - p}{\hat{\rho} - \rho}} \qquad \hat{W} = \sqrt{\frac{\rho}{\hat{\rho}}\frac{\hat{p} - p}{\hat{\rho} - \rho}} \tag{26}$$

Hence follows

$$W - \hat{W} = \sqrt{(\hat{p} - p)\left(\frac{1}{\rho} - \frac{1}{\hat{\rho}}\right)} \tag{27}$$

The velocity difference $W - \hat{W}$ is the same for all reference systems moved perpendicularly to the shock front, and is hence equal to the velocity W at the contact surface in figure 8. With the designations used there, we have therefore exactly:

$$W^2 = (p - p_1)\left(\frac{1}{\rho_1} - \frac{1}{\rho}\right) = \frac{p}{\rho_1}\left(1 - \frac{\rho_1}{\rho}\right)\left(1 - \frac{p_1}{p}\right) \tag{28}$$

Equations (24) and (25) may be interpreted as two relationships for the unknown state at the contact surface, W and p/p_1. W can be easily eliminated, and we obtain

$$\frac{c_1}{c_0} = \frac{2\sqrt{\kappa_1}}{\kappa - 1}\sqrt{\frac{p_1}{p}}\left[1 - \left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{2\kappa}}\left(\frac{p_1}{p_0}\right)^{\frac{\kappa-1}{2\kappa}}\right] \tag{29}$$

which is an equation for the required sonic-velocity ratio of low-pressure and high-pressure gas in the initial state when the corresponding pressure ratio p_1/p_0 and the pressure ratio in the shock p/p_1 are given. The two ratios of the specific heats in the initial state, κ_1 and κ, also enter into equation (29). It is assumed that the high-pressure gas expands like an ideal gas of constant specific heat; the compression shock, however, may be connected with dissociation, ionization, or merely with variations of the specific heats if it is only sufficiently strong.

Since the pressure sharply decreases even in the high-pressure gas, a pressure ratio p_0/p_1 as large as possible is necessary to guarantee sufficient pressure rise in the shock. A pressure of $p_0 = 300$ atm on the high-pressure side and a vacuum of $p_1 = 0.03$ atm on the low-pressure side are not hard to achieve in the laboratory for small tube dimensions. If we provide, furthermore, a pressure ratio in the shock of $p/p_1 = 280$, which according to table 2 corresponds to a Mach number $M = 14.8$, and assume $\kappa_1 = \kappa = 1.40$, we can calculate from equation (29)

$$\frac{c_1}{c_0} = \frac{5\sqrt{1.40}}{\sqrt{280}}\left[1 - \left(\frac{280}{10,000}\right)^{1/7}\right] = 0.142 = \frac{1}{7}$$

If we use H_2 as the propellant for the compression of air, the required temperature ratio then is, with equation (3)

$$\frac{T_0}{T_1} = \frac{m_0}{m_1}\frac{c_0^2}{c_1^2} = \frac{2}{29} 7^2 = 3.4$$

If, therefore, the air is maintained at room temperature $T_1 = 288^\circ$, a heating of the hydrogen to $T_0 = 980^\circ$ (= 600° C) is necessary, to produce the conditions of table 2 at $M = 14.8$. These are not particularly large demands on test technique.

The calculation contains three further inaccuracies which could slightly modify the result. A heating of hydrogen leads to a small decrease of κ, which could become noticeable in equation (29) because $\kappa - 1$ enters. Furthermore, according to table 2, the air is compressed from 0.03 times to 0.30 times normal density. This leads to a somewhat higher dissociation and a somewhat lower temperature but not to any essential changes. Finally, the absolute temperature ahead of the shock is assumed to be 288°, thus 30 percent higher than in table 2, which results in a higher temperature behind the shock and correspondingly higher dissociation. This effect probably overbalances those named first.

Therefore we produce, in the present case, a temperature of more than $5,000^\circ$ abs in the shock tube, thus approximately the temperature of the surface of the sun. In contrast, the temperature in the adjacent H_2 is very low, namely

$$\frac{T_{H_2}}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} = \left(\frac{280}{10,000}\right)^{1/7} = 0.36 \quad \text{or} \quad T_{H_2} = 350^\circ \text{ abs}$$

This reproduces a quite general property of explosions. In chemical combustion, as occurs for instance in powder, the temperature does not rise beyond $2,500^\circ$ to $3,000^\circ$, since then, as in the case of hydrogen and oxygen, the dissociation becomes so strong that it consumes all further available energy. The strong pressure drop in the explosion leads to a considerable cooling of the powder gas; the dissociation need by no means find the time to recombine. The air, in contrast, shocklike compressed by the explosion, is extraordinarily heated up and represents, further along, the hot part of the process.

The possibilities for producing high temperatures in the shock tube are not exhausted with what we have described above. When the shock, in figure 9, finally arrives at the right end of the tube, the thermodynamic state is further pushed up by reflection. A narrowing of the tube toward the right likewise leads to energy concentration and to increases of temperature and pressure. Regarding these facts, or the use of other gases, we refer for instance to the reports of Kantrowitz (ref. 6).

The achievement of such extreme states with relatively small effort must of course be paid for with a disadvantage: the transient nature of the phenomenon. The phenomenon which is of interest extends only from the front of the shock to the surface of contact with the high-pressure gas, thus only over a fraction of the entire shock tube, and it races by with a speed of (eq. (25))

$$W = \frac{1}{\sqrt{\kappa_1}} \sqrt{\frac{p}{p_1}}\, c_1 = \sqrt{\frac{280}{1.40}}\, 340 \text{ m/sec} = 4,800 \text{ m/sec}$$

If we succeed, therefore, in extending the observed state to 0.50 m in a shock tube of 5 m length, it lasts at one point 1/10,000 sec. This is sufficient for photographic recording. Subjective observation is of course impossible.

5. RAMJET POWER PLANT

In the face of the unusual conditions encountered at high speeds, there comes to mind the question as to what speeds are altogether attainable with airplanes or other flying bodies. We choose as the starting point the thrust equation for a free-flying body. As the loss of momentum leads, on a surface, to a pressure rise, to a force in the flow direction and, in the case of a body, in the last analysis to a drag (shown for instance in eq. (15)), so the production of momentum leads to a thrust S. In what follows, we shall consider the simplest and most important case in practice: that the production of momentum occurs in a region of constant pressure (fig. 10). It is unimportant whether pressure differences appear on the thrust body itself. Let the pressure be constant only in a certain region characterized by the rectangular boundary of the body. Let us consider there the entering jet with the cross section f_∞, the velocity W_∞, and the quantity G_∞ flowing through per unit time, and the exhaust jet with the corresponding quantities without subscript.

The loss of momentum in a certain direction, either by stopping or by rectangular deflection, produces a force $K = \rho W f \times W = GW$; the production of momentum generates the corresponding thrust. Thus there results for the total thrust

$$S = GW - G_\infty W_\infty \tag{30}$$

In many cases a definite inflowing mass is used for increasing the velocity; this applies exactly for all propellers and with very good approximation for all jet power plants. Then G is equal to the quantity G_∞ coming from the infinite approach-flow region and

$$S = G_\infty(W - W_\infty) \tag{31}$$

is valid.

In the case of the rocket, however, which will be treated in the next section, the momentum is imparted to a mass which is carried along, $G_\infty = 0$, and

$$S = GW \tag{32}$$

is valid.

The ramjet power plant operates as follows: A part G of the approaching air is given an increased pressure by being stopped. In this state of rest the air is heated whereby its density greatly decreases. The heated air finally is expanded again to the initial pressure. The velocity W attained by the air in the pressure gradient is larger than the velocity W_∞ lost by the air during the pressure rise of the stopping because the air density during the expansion is smaller. For the lighter the air, the greater is its acceleration in a given pressure gradient.

This type of propulsion was discovered by the Frenchman Lorin at a time when flying was still done at low subsonic speeds. For supersonic velocities, this type of propulsion was rediscovered by Trommsdorff in the early years of the last war. As will be shown directly, this type of propulsion is of eminent importance precisely for supersonic speeds. In order to understand this, one has only to start from the principle that the efficiency η_c of an ideal thermodynamic engine is given, according to Carnot, by the ratio of temperature rise between heat input and heat exhaust to the temperature of the heat input. The heat input takes place at the stagnation temperature T_0. Here as well as for the gasoline engine, the cooling of the working gas is replaced by an exchange of the hot air blown off in favor of newly supplied cold fresh air. With the equation (11), applied to the approach-flow state T_∞, M_∞, there follows thus for Carnot's efficiency

$$\eta_c = \frac{T_0 - T_\infty}{T_0} = 1\Bigg/\left(1 + \frac{2}{\kappa - 1}\frac{1}{M_\infty^{\,2}}\right) \tag{33}$$

The approximation of an ideal gas of constant specific heat, assumed for equation (11), is sufficiently satisfied for the following considerations. Table 4 gives a few results.

Whereas, therefore, the efficiency of the ideal ramjet power plant is still low for flights at the speed of sound, it equals, for $M_\infty = 3$, a diesel engine working ideally with a pressure of 37 atm! With increasing flight Mach number M_∞ the efficiency improves still further. Thereby this type of propulsion proves to be the ideal engine for steady supersonic flight. Of course, some losses are to be expected for this type of propulsion, too; among them, the kinetic energy being lost with the thrust jet is by far the most important. Thus only part of the performance given by Carnot's efficiency is used "profitably." To an observer standing on the ground and considering the body flying at W_∞, the work performed per unit time against the drag forces or against gravity is

$$W_\infty S = GW_\infty(W - W_\infty) \tag{34}$$

The exhaust jet, however, discharges an amount of kinetic energy per unit time given by

$$G\,\frac{(W - W_\infty)^2}{2} \tag{35}$$

The discharged heat energy need no longer be taken into consideration. This energy has already been represented by Carnot's efficiency. The ratio of the desired mechanical output and the sum of the two actually achieved mechanical outputs yields the jet efficiency

$$\eta_{st} = \frac{2W_\infty(W - W_\infty)}{2W_\infty(W - W_\infty) + (W - W_\infty)^2} = \frac{2W_\infty}{W + W_\infty} \tag{36}$$

The product of the two efficiencies η_{st} and η_c then gives rather accurately the efficiency which has to be taken into account in practice. An example will clarify the possibilities.

With assumption of isentropic (adiabatic) expansion it is easy to derive from equation (11) the generalized Bernoulli equation

$$W^2 = 2c_pT_0\left[1 - \frac{T}{T_0}\right] = 2c_pT_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right] \tag{37}$$

which is similar to equation (24), yet essentially different from it. At a certain pressure ratio p/p_0, W is proportional to $\sqrt{T_0}$ which is a square-root relationship between W, c, and T appearing in different forms. If in the case $M_\infty = 3$ the stagnation temperature is quadrupled by heating, which leads to a temperature of 2,536° abs, the exhaust velocity is doubled: $W = 2W_\infty$. We then arrive at efficiencies of

$$\eta_{st} = 2/3 \qquad \eta = \eta_c \eta_{st} = 0.43$$

thus at very promising values. Of course, the jet efficiency could be improved by less heating. However, the thermodynamic efficiency alone is not the governing factor for the designer. Above all, a certain thrust S is necessary.

If the problem consists, for instance, in flying at constant speed, in steady flight, for a long distance, say across the Atlantic Ocean, the thrust S is equal to the drag D or with equations (31) and (16)

$$G_\infty(W - W_\infty) = f_\infty \rho_\infty W_\infty (W - W_\infty) = c_D F \rho_\infty {W^2}_\infty/2$$

Thereby the ratio of the cross-sectional area of the oncoming jet f_∞ and the frontal area of the body F is

$$\frac{f_\infty}{F} = \frac{c_D}{2} \frac{W_\infty}{W - W_\infty} \tag{38}$$

The drag coefficient c_D of a flying vehicle lies far below that of blunt bodies (fig. 5). If it is assumed to be $c_D = 0.30$ (referred to the frontal area, not, as in the case of airplanes, to the wing area or the like), 15 percent of the frontal area is required for the oncoming jet at $W = 2W_\infty$; this appears reasonable.

It is true that a reduction of the difference $W - W_\infty$ would lead to an increase of η_{st}; but at the same time it would increase the jet area f_∞ required according to equation (38), which is undesirable. The designer therefore demands a certain "thrust concentration" and foregoes an excessive increase of the jet efficiency.

It is one of the small inaccuracies of the present representation that we always speak of heating but that, in practice, this heating takes place by combustion of a hydrocarbon. An input of mass occurs which is, however, rather insignificant. For instance, the combustion of gasoline and benzol in air under constant pressure in the mass ratio 1:100 yields a temperature rise of approximately 400°. As for the rest, a part of the smaller neglected effects acts favorably.

With the quadrupling of the stagnation temperature at $M_\infty = 3$ the boundary is reached at which the dissociation of the air sets in. There would not be any objection against the storing of energy in the form of dissociation if this energy would become free again during the cooling off of the jet in the expansion. This would even have the advantage that the combustion-chamber temperature would not rise above a certain limit. The control of high combustion-chamber temperatures represents, anyway, one of the serious technical problems. However, the time available during the expansion is too short for recombination of the dissociation and adjustment to the cooler thermodynamic equilibrium state since a stronger cooling occurs only at very high speeds. Thereby a considerable part of the energy going into dissociation is lost.

The temperature up to which heating may be continued in the combustion chamber is thus limited at least by the onset of dissociation - unless the strength of the combustion-chamber walls (essentially a cooling problem) imposes lower limits. This means a barrier for the heat supply which restricts application of the ramjet power plant by a limiting Mach number. Let us assume the temperature of 2,536° abs - an enormous heat - to be the upper temperature limit in the combustion chamber. Then there results for $M_\infty = 5$ an exhaust velocity of $W = \sqrt{2,536/1,338}\, W_\infty = 1.38W_\infty$ compared to $W = 2W_\infty$ for $M_\infty = 3$. This means a considerable increase of the area ratio f_∞/F, even though a small reduction of c_D with M_∞ may be assumed.

However, temperature problems do not occur merely in the combustion chamber but on the entire body of the flying vehicle. It is shown that the stagnation temperatures given by equation (11) and calculated in table 4 are attained not only by ordinary storing but also - almost - by the stopping of the air due to friction at the surface of the flying body. Even though these temperatures lie far below the heating temperatures in the combustion chamber, the related problems are not any smaller since enormous difficulties are encountered in the cooling of larger parts of a flying body. If the vehicle is manned, a very radical cooling of the cockpit is needed already for $M_\infty = 3$. The electronic components of remotely controlled bodies also are very sensitive to temperature; they must be cooled or strongly heat-insulated. The simplest solution depends essentially on the length of the flying time. In the case of wing constructions, the strength properties of the metals must be exploited to the utmost. The metals greatly vary in strength with the temperature. Aluminum melts at 660° C. Due to its higher melting point of 1,530° C, soft iron is harder than steel at high temperatures. In finer constructions, however, not only the general strength must be considered. The heat transfer to the wing and the heat conduction in the wing may lead to a nonuniform temperature distribution and to a warping of the wing.

Still another difficulty must be pointed out which lies in the appearance of high centrifugal acceleration. If the latter is designated by g and the radius of curvature of the path by R, the following relationship, which is known from mechanics, applies:

$$gR = W^2 \tag{39}$$

For a speed of $W = 1{,}000$ m/sec, thus approximately $M = 3$, and a path radius of $R = 100\ \text{km} = 10^5$ m, one arrives at a centrifugal acceleration of 10 m/sec^2, thus approximately at the acceleration of gravity. If $4g$ is regarded as the maximum admissible acceleration for the human body, only path radii larger than 25 km are admissible for $M = 3$. The pilot must turn in a circle of this radius if he misses his target. For the wings and control components of a pursuit rocket, the intended path radii thus are sometimes much more important than the production of lift for overcoming gravity.

All this shows that the main difficulty in supersonic flight does by no means lie in the production of the required forward thrust, but in the control of the temperatures and centrifugal forces.

6. ROCKET

In a well-known and very good book for young readers, written in 1931, on various philosophical, mathematical, and technical problems there is the sentence: "It can easily be calculated that the upper limit to which present fuels can take us, is about 400 km above the surface of the earth . . ." The corresponding calculation is not given; but, doubtlessly, it is based on the relationship that the energy required for lifting a mass $\bar{M}$ ($\bar{M}$ for the mass in contrast to M for the Mach number) to the height H can at most be equal to the reaction energy or combustion energy which this mass may contain. At the anticipated height of 400 km - compared to the size of the earth's radius of 6,380 km - the gravitational acceleration g may be regarded as constant. With q as the reaction energy per unit mass there results, in this manner, the energy equation: force $\bar{M}g$ times distance H equals mass times reaction heat q. The mass may be cancelled from this equation with the result

$$gH = q \tag{40}$$

A good powder has about $q = 1{,}000$ cal/g or, according to equation (17), $q = 4.186\ (\text{km/sec})^2$. With $g = 10\ \text{m/sec}^2 = 10^{-2}\ \text{km/sec}^2$, equation (40) leads to the height of rise $H = 418.6$ km.

As to the reaction energy, one could of course raise the objection that much greater heights could be attained with liquid-fuel rockets (see for instance the oxygen-methane reaction in table 6) and that, more recently, atomic energies open up completely different possibilities. However, we shall not enter into details about this since the expression (40) itself is based on a wrong assumption: that the reaction energy of a mass is consumed in lifting or else for accelerating one and the same mass. The processes have been sufficiently discussed in the preceding section to understand them correctly in what follows.

The thrust of a rocket is given by equation (32); W is the "exhaust velocity," the velocity in the rocket jet relative to the rocket body. After all the propellant has been ejected, the energy balance is such that the thermal energy of the propellant has been used to impart to the rocket body a certain kinetic or potential energy. Undesirably, mechanical energy (kinetic or potential) has, in addition, been given to the propellant and possibly to masses released or left behind in flight. The distribution of the mechanical energy between the masses involved is rather nonuniform and may, theoretically, be completely in favor of the rocket.

Let us assume, for example, that the acceleration of the rocket occurs horizontally - or also vertically but at sufficiently slight differences in height - so that the variations of the potential energy of the masses under consideration do not play any part. Let us assume, furthermore, that the exhaust velocity W of the propellant jet can be regulated and is, in each case, adjusted so that it is equal to the flight velocity of the rocket; then, the ejected propellant mass is at rest, relative to the observer. In this ideal case the entire thermal energy of the propellant mass $\bar{M}_T$ is used for imparting a kinetic energy to the rocket end mass $\bar{M}_R$.

It is true that the "thermal energy" there is not identical with the "reaction energy" at a particular temperature, as is usually indicated in tables (cf. table 6). The variations in the internal energy of the propellant also play a certain role. For the considerations of this section, however, which have only the purpose of orientation, these variations are unimportant. But if compressed air is used as the "rocket propellant," the "thermal energy" consists exclusively in variations of "internal energy." If thus the entire thermal energy per unit mass of the propellant which has been released is denoted by q and V is the flight velocity of the rocket, there applies in the ideal case

$$\frac{V^2}{2}\bar{M}_R = q\bar{M}_T \tag{41}$$

By selecting a sufficiently small mass ratio $\bar{M}_R/\bar{M}_T$ we can therefore impart to the rocket an energy far exceeding equation (40). Evidently equation (41) defines an unattainable ideal case whose value consists, above all, in establishing a limit for the most favorable conditions.

Strictly speaking, the exhaust velocity W must not be set equal to the flight velocity, at least at the beginning. At the start a W, even though a minimum W, must exist so that a starting thrust may be present. Thus we deal in this respect, as well, with a theoretically constructed limiting case.

In spite of all the defects inherent in the ideal case investigated, it teaches clearly a few fundamental principles. Extreme end velocities require large masses of propellant. Empty fuel containers will have to be dropped, wherever possible, in order not to accelerate unnecessary dead weight. The most favorable exhaust velocity W lies below the end velocity of the rocket. Thus the rocket jet has, at the start, absolute velocities which are directed against the flight direction. During the flight, the absolute velocities of the jet decrease more and more, vanish, and are directed in the direction of flight at the end of the combustion time. In this manner a minimum of mechanical energy may be lost to the propellant.

The rocket combustion chamber of constant exhaust velocity W represents the case which is by far the most important in practice. We shall compare this frequently treated case (ref. 7) with the ideal case. Let U and $\bar{M}$ be the velocity and mass of the rocket during the combustion. The thrust then imparts to the mass an acceleration dU/dt and there applies according to Newton

$$\bar{M}\,\frac{dU}{dt} = GW \qquad (42)$$

an equation which is in terms of velocities. The propellant mass ejected per unit time equals the decrease of the rocket mass

$$-\frac{d\bar{M}}{dt} = G \qquad (43)$$

Eliminating dt from equations (42) and (43), we can relate the variation $d\bar{M}$ directly to the variation dU

$$\frac{dU}{W} = -\frac{d\bar{M}}{\bar{M}}$$

For W = Constant, this can be easily integrated

$$-\frac{U}{W} = \ln \bar{M} + \text{Constant}$$

and since for the beginning and the end of the acceleration

$$U = 0: \quad \bar{M} = \bar{M}_R + \bar{M}_T,$$

$$U = V: \quad \bar{M} = \bar{M}_R$$

is valid, there follows finally

$$\frac{V}{W} = \ln \frac{\bar{M}_R + \bar{M}_T}{\bar{M}_R} \quad \text{or} \quad \frac{\bar{M}_R + \bar{M}_T}{\bar{M}_R} = e^{V/W} \tag{44}$$

The fraction $\left(\bar{M}_R + \bar{M}_T\right)/\bar{M}_R$, that is, the starting mass of the rocket divided by its final mass, is denoted as mass ratio. According to equation (44), this ratio must assume the value $e = 2.718$ ($\ln e = 1$) if the rocket is to attain the exhaust velocity W of the propellant, unimpeded by air drag or gravity. For determination of the velocity with the smallest jet losses, the kinetic energy of the rocket $\left(\frac{1}{2}\right)\bar{M}_R V^2$ would have to be related to the kinetic energy of rocket and propellant in the final stage. Thus we must sum over the kinetic energy of the various parts of the jet. Without demonstrating here this comparatively elementary calculation, we shall only report that the highest value of the thus defined jet efficiency η_{st} results for a mass ratio of 5. We obtain approximately

$$\left.\begin{array}{lll} \left(\bar{M}_R + \bar{M}_T\right)/\bar{M}_T = 5 & V/W = 1.6 & \eta_{st} = 0.65 \\ \left(\bar{M}_R + \bar{M}_T\right)/\bar{M}_T = 2.72 & V/W = 1.0 & \eta_{st} = 0.58 \end{array}\right\} \tag{45}$$

In the second line, the values for $V = W$ are given. Thus the ideal case, given by equation (41), is not so very extraordinary, after all. With the usual rocket of constant exhaust velocity W and ejection mass G, more than 50 percent of the ideal case can be attained. The most serious difficulty lies rather in the achievement of the required mass ratio. The construction of a rocket, the propellant mass of which amounts to 63 percent ($V/W = 1.0$) or even 80 percent ($V/W = 1.6$), represents an enormous design problem. For the V2, the approximate value $\left(\bar{M}_R + \bar{M}_T\right)/\bar{M}_R = 3.5$ is given in reference 8. Only the smallest part of the end mass $\bar{M}_R$ of the rocket, however, consists of the useful load to be transported; the largest part is occupied by the mass of the fuel tanks, of the rocket-combustion chamber, and of the various necessary accessories.

For judging the difficulties of building interplanetary rockets, we shall first calculate the necessary starting velocities. We assume that the body is directly shot away from the earth's surface into a vacuum. Actually, the rocket is driven comparatively slowly through the lower atmosphere, the troposphere, in order to keep the losses by air drag low. Outside the troposphere, the rocket is then rapidly accelerated to the final velocity V. However, since not only the troposphere, with approximately 10 km height as a region of possible friction losses, but also the height required to attain V is small compared to the distances considered, we may calculate as if the body were entering the vacuum with V directly from the earth's surface.

The gravitational acceleration g decreases quadratically with the distance R from the center of the earth. On the surface of the earth, it is $R = R_0 = 6{,}380$ km, $g_0 = 9.81$ m/sec^2. Thus

$$g = g_0\left(\frac{R_0}{R}\right)^2 \tag{46}$$

is valid. If we now assume that a body rotates as a satellite, at the distance R from the center of the earth, with the velocity V_s, the relationship (39) between path radius, centrifugal acceleration, and velocity is valid, so that

$$V_s^2 = g_0 \frac{R_0^2}{R} \tag{47}$$

Table 5 shows numerical values and the Mach numbers referred to a sonic velocity of $c = 330$ m/sec. At such speeds a body does, therefore, not need wings. The path curvature produces the required "lift."

If a body is to fly at the distance R with V_s, its starting velocity V must be so large that the decrease of the kinetic energy is equal to the work performed against the gravitational pull. All these quantities are proportional to $\bar{M}$; for this reason, the mass is cancelled from the calculation:

$$\frac{V^2}{2} - \frac{V_s^2}{2} = \int_{R_0}^{R} g_0 \frac{R_0^2}{R^2}\, dR = g_0 R_0 \left(1 - \frac{R_0}{R}\right)$$

or with equation (47)

$$\frac{V^2}{2} = g_0 R_0 \left(1 - \frac{1}{2}\frac{R_0}{R}\right) \tag{48}$$

The kinetic energy required for escaping the field of gravity of the earth ($R \rightarrow \infty$) is therefore twice the kinetic energy of a satellite of equal weight at a small distance from the surface of the earth. For the former, we obtain with equation (17)

$$\frac{V^2}{2} = g_0 R_0 = 63(\mathrm{km/sec})^2 = 15{,}000 \ \mathrm{cal/g}$$

This quantity of energy is not unattainable. The oxygen-methane reaction, for instance, yields approximately 1/6 of it (table 6). Thus energies are concerned which are perfectly producible. However, the velocities to be reached are of astronomical dimensions as is shown by a comparison of tables 1 and 5 - a hint that the movements of the stars must be traced back to thermal energy sources in atomic reactions, not to chemical reactions.

Table 6 shows several reaction heats and the mass ratio for the ideal case, equation (41). The melting of a body does not change anything in its molecular composition and relatively little in its structure; not even the density changes essentially. It is therefore understandable that the "reaction heat" connected with melting is generally much smaller than that connected with a chemical reaction. The oxygen-hydrocarbon reaction O_2-methane, the detonating-gas reaction, and the combination of atomic to molecular hydrogen are given as a representative for the last-named reaction. It corresponds, of course, only to a thermo-chemical equation, not to a practical process, since the atomic hydrogen is not stable under "normal" thermal conditions. Nevertheless, this hypothetical process is very useful for the following consideration.

In regard to the structure of matter, chemical reactions are changes in the shell of the electrons. In the case of light elements and light molecules where the electron shell surrounds only few protons and neutrons, such changes may require the same energy as in the case of heavy elements. Thus, very much higher reaction heats are possible when substances of low molar weight participate than in the case of substances of high molar weight m.

Finally we have the heat quantity of an atomic reaction of $\delta = 0.005$ mass defect which, of course, amounts to an extraordinarily strong atomic reaction. The heat set free is equal to the mass defect times the square of the speed of light. The velocity corresponding to the equivalent kinetic energy is thus equal to the speed of light multiplied by $\sqrt{2\delta}$. Changing the structure of the atoms leads to energy transformations which are incomparably larger than those of chemical processes. Of course, for atomic reactions much more than for chemical reactions, it is always true that only part of the propellant represents really active reacting substance while another part is carried along as "dead substance." Table 6 is not supposed to reflect more than a picture of the possibilities.

It becomes clear that even the mass ratios of 6.9 and 5.7 calculated for the ideal case (eq. (41)), required for surmounting the gravity pull, are extraordinarily high for a hydrocarbon and a detonating-gas reaction and pose probably unsolvable problems to the designer. (As was emphasized before, the reaction heat in table 6 and q in equation (41) are only approximately equal.) The pertaining energies expressed in velocity squares show that these values also lie throughout considerably below the required values of V given in table 5.

It is instructive to look somewhat more closely at the conditions, for instance, in the case of a hydrocarbon reaction. From the known relationships between the specific heats

$$c_p - c_v = R/m \qquad c_p/c_v = \kappa$$

if follows readily that

$$c_p = \frac{\kappa}{\kappa - 1}\frac{R}{m} \tag{50}$$

Since in the O_2, CH_4 reaction the number of molecules is maintained, the molar weight is not changed, either, by the process and lies, with $m = 27$, close to the value of air. With $\kappa = 1.30$, equation (50) leads with $R = 2$ cal/(g degrees) to a value of approximately $c_p = 0.32$ cal/(g degrees). Thus the reaction would result in a temperature increase of

$$\Delta T = \frac{q}{c_p} = \frac{2{,}560}{0.32} = 8{,}000^\circ.$$

On the one hand, the temperature would be completely intolerable; on the other, it is never reached, due to dissociation setting in, as shown by the example for air calculated at the beginning in connection with the flight of a meteor. These enormous energy transformations in the rocket combustion chamber are therefore not even desirable and the carrying along of "dead substance" is quite welcome. The maximum velocity attainable in the jet can be easily given from equation (37) under the assumption of pressure drop to vacuum $p \to 0$. With equation (3), W_{max} is

$$W_{max} = \sqrt{2c_pT_0} = \sqrt{\frac{2}{\kappa - 1}}\, c_0 \tag{51}$$

The maximum velocity attainable in the steady pressure gradient, expressed by equation (51), is thus considerably smaller than that of an unsteady two-dimensional explosion, equation (22). There exists, also, an essential difference between the two processes. Whereas in the case of the steady phenomenon every particle is equal and gradually assumes in the pressure gradient $p \rightarrow 0$, the velocity of equation (51), only a small part of the matter involved is distinguished by attaining the velocity of equation (22) for $p \rightarrow 0$ in the case of the two-dimensional explosion. Furthermore, it must be noted that the flow velocity in the unsteady two-dimensional explosion exceeds the flow velocity of the steady flow only in the case of a high pressure gradient. The pressure gradient for which both velocities are equal can be easily determined by equating W/c_0 according to equation (24) and according to equation (37). We obtain

$$\ln \frac{p_0}{p} = \frac{2\kappa}{\kappa - 1} \ln \frac{\kappa + 1}{3 - \kappa}$$

or for $\kappa = 1.40$

$$\frac{p_0}{p} = 17.1$$

The speed of sound depends essentially on the molar weight and thus, for the mixture investigated, does not differ very much from the value for air at the same temperature. If we assume, in consideration of material and dissociation, an absolute temperature of $3,000^{\circ}$ abs, we have approximately $c_0 = \sqrt{10} \times 350$ m/sec = 1.1 km/sec and according to equation (51), with $\kappa = 1.3$, the exhaust velocity $W = W_{max} = 2.8$ km/sec. This value, however, lies far below the ideal value. For leaving the region of the pull of gravity, according to table 5 and equation (45), $W = 0.62 \times 11.2$ km/sec = 7 km/sec would be desirable. Somewhat more than $W = 2$ km/sec was attained with the V2 during the first years.

Hence, it follows that substances of low molar weight are highly desirable not only as reacting masses but also as dead rocket-jet masses for extreme speeds of travel. Due to their high sonic velocity, they also make high exhaust velocities possible, according to equation (51). The transformation of higher energy quantities in the combustion chamber, in contrast, does by no means lead to higher temperatures, compared to heavier substances, because, according to equation (50), the specific heat is much higher for smaller m. Neither is the tendency toward dissociation at all stronger for gases of small molar weight (Cf., for instance, H_2 according to reference 9.)

The application of an atomic jet, in spite of the enormous energy transformations connected with it, does not immediately lead to a solution of the difficulties: Exhaust velocities far exceeding the flight velocity mean an enormous waste of energy. However, use of atomic energy as an energy source for the heating of a dead substance of low molar weight is to be regarded as a solution promising progress. This dead substance should, of course, be carried along as a fluid or solid matter in order to keep the container volumes tolerable. For this reason it is very regrettable that H_2, as the lightest substance by far, has a boiling point of -253° C and, as a fluid, in addition, has very low density.

A possibility of achieving the necessary mass ratios for a flight into interstellar space or for sending out an artificial earth satellite consists in shooting a "daughter rocket" from a "mother rocket" and possibly repeating this procedure in several stages. It is evident that the greatest part of the propellant energy utilized must be used to accelerate the entire vehicle and only a small part can be used to the advantage of the basic purpose. Thus, it will have to be regarded as an extraordinary technical performance if we succeed, within the next few years, in shooting a satellite of 50 kg weight into an altitude of several hundred km. Within a foreseeable time, the shooting of space rockets will surely not become such a familiar phenomenon as the penetrating of meteors into the earth's atmosphere treated at the beginning. Certainly it will always be a very costly procedure; it is surely extremely optimistic to insert for the actual pay load an expenditure of 10 times the amount calculated in connection with equation (48), thus 150,000 cal/g. With this digression to the questions of interplanetary aviation we shall close the section on rockets.

7. RÉSUMÉ

All problems treated have in common the high-temperature differences connected with high supersonic speeds - a consequence of the large kinetic energies inherent to such flows. High temperatures appear, therefore, when the air in front of meteors and on fast-flying bodies is stopped. High temperatures are necessary when high speeds are to be produced in propulsive jets of flying bodies. In increasing the temperature beyond several thousand degrees, the heating is greatly reduced by dissociation and ionization phenomena since a considerable part of the energy is used for splitting up the molecules instead of for increasing the temperature. The influence of the molar weight is very great. The light gases, like hydrogen, with the considerably higher flight velocities of the molecules, permit the achievement of high flow velocities more readily than gases of high molar weight. With hydrogen as the propellant it is therefore possible to produce, in the laboratory, transient high temperatures in air, and gases of low molar weight appear particularly suitable as the propellant-jet substance for rockets. With them, the astronomical velocities which are necessary for space rockets are most nearly attainable.

The high centrifugal forces, which balance the gravity pull at the high velocities in space, make themselves clearly felt even at supersonic speeds. They impose a limitation on the path curvatures of flying bodies.

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National Advisory Committee
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TABLE 1.- ASTRONOMICAL SPEEDS

	km/h	km/sec
Velocity of sound c in the stratosphere (temperature -50° C)	1,080	0.30
Circumferential velocity of the earth at approximately 50° latitude	1,080	0.30
Velocity of revolution of the moon about the earth		1
Velocity of revolution of the earth about the sun		30
Velocity of the sun with respect to the next constellation		20
Velocity at the edge of the universe		50,000
Velocity of light		300,000

TABLE 2.- COMPRESSION-SHOCK WAVE FOR $\hat{\rho}$ = NORMAL DENSITY

$\hat{T}$, abs	$R\hat{T}$, [cal]	$\hat{e}m$, [cal]	$m/\hat{m}$	$\hat{\rho}/\rho$	Flight altitude, [km]	$\hat{T}/T$	$\hat{p}/p$	M
						(for $T = 223°$)		
1×10^3	2×10^3	5×10^3	1	6.0	14.3	4.5	27	4.8
5×10^3	10×10^3	56×10^3	1.20	10.4	17.8	22.4	280	14.8
10×10^3	20×10^3	235×10^3	1.98	12.9	19.2	44.8	1140	29.8
20×10^3	40×10^3	500×10^3	2.48	11.1	18.2	89.6	2460	44.1

TABLE 3.- COMPARISON OF THE ENERGY FOR VERTICAL PENETRATION OF THE ATMOSPHERE

Kinetic energy per unit mass: $\frac{W^2}{2}$

Resistance work per unit mass: $\frac{94}{r\,[\mathrm{cm}]}\,\frac{W^2}{2}$

Heat of fusion of iron: $49\left[\mathrm{cal/g}\right] = 0.41\left[\frac{1}{2}(\mathrm{km/sec})^2\right]$

TABLE 4.- CARNOT EFFICIENCY η_c, CORRESPONDING PRESSURE RATIO OF THE PISTON ENGINE, AND STAGNATION TEMPERATURE OF THE RAMJET POWER PLANT

M_∞	η_c	p_0/p	T_0/T_∞	$T_\infty = 223^\circ$ abs	
				T_0	$T_0 - 273$
1	1/6	1.9	1.20	268° abs	-5° C
3	9/14	37	2.80	634° abs	361° C
5	5/6	530	6.00	1,338° abs	1,065° C

TABLE 5.- STARTING AND SATELLITE VELOCITIES FOR VARIOUS DISTANCES FROM THE EARTH'S SURFACE

$R - R_0$, [km]	V_s, [km/sec]	V, [km/sec]	M for c = 330 m/sec
0	7.9	7.9	24
500	7.3	8.2	25
1,000	6.8	8.5	25.5
2,000	6.0	8.9	27
∞	0	11.2	34

TABLE 6.- REACTION HEATS

	q, [cal/g]	q, $\left[\frac{1}{2}(\text{km/sec})^2\right]$	$\frac{\bar{M}_R + \bar{M}_T}{\bar{M}_R}$
Heat of fusion of iron	49	0.64^2	
Black powder	630	2.3^2	25
O_2 and CH_4	2,560	4.6^2	6.9
O_2 and $H_2 \rightarrow H_2O$	3,220	5.2^2	5.7
$2H \rightarrow H_2$	49,200	20.2^2	1.3
1/2 percent mass defect		$30,000^2$	

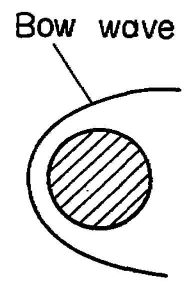

Figure 1.- Bow wave for high Mach number.

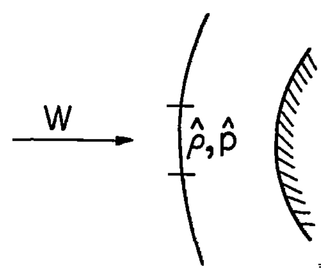

Figure 2.- Bow wave in the stagnation region.

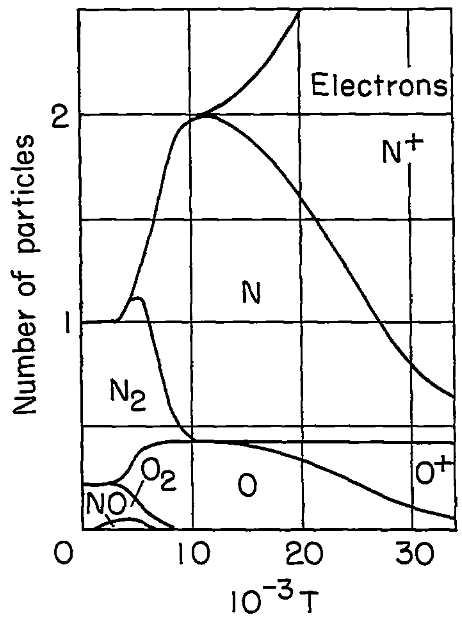

Figure 3.- Composition of the air for normal density, as a function of the temperature (according to G. Burkhardt (ref. 2)).

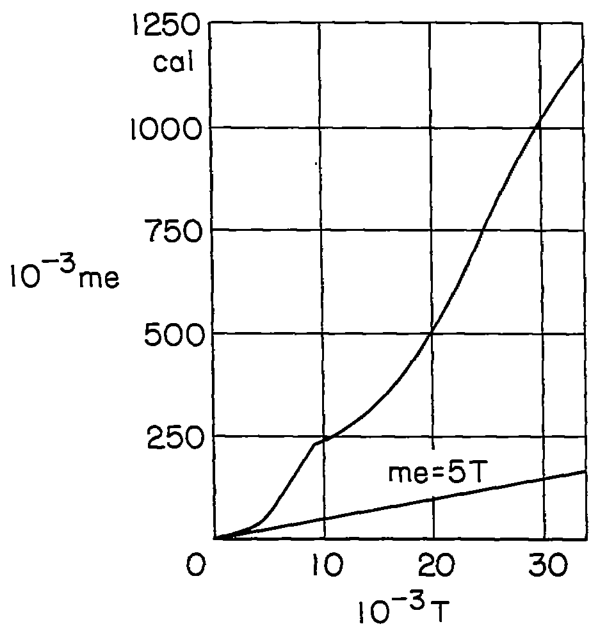

Figure 4.- Internal energy of $m = 29g$ air for normal density, as a function of the temperature (according to G. Burkhardt (ref. 2)).

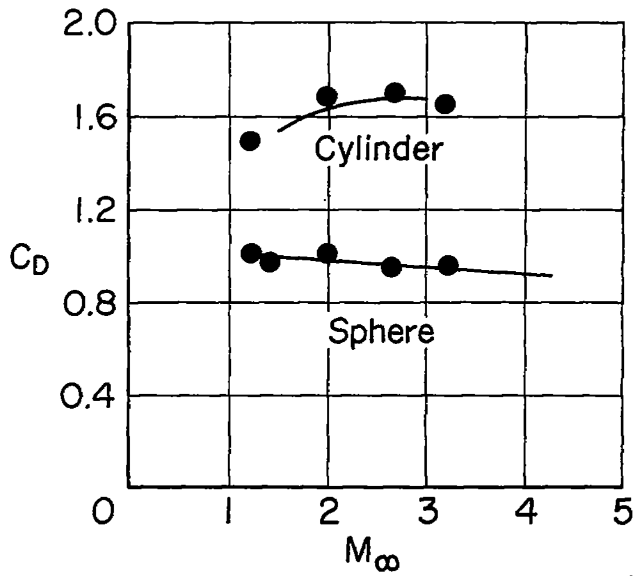

Figure 5.- Experimental drag coefficients for a sphere and a cylinder in axial flow (according to ref. 5).

——— Peenemünde measurements

• • Göttingen measurements

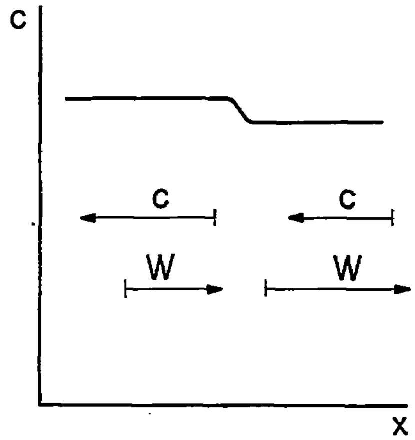

Figure 6.- Change of state in a sound wave.

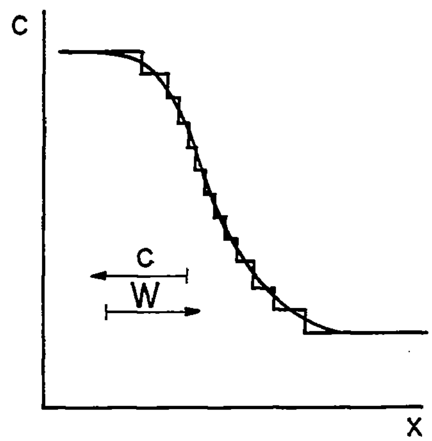

Figure 7.- Wave of finite amplitude.

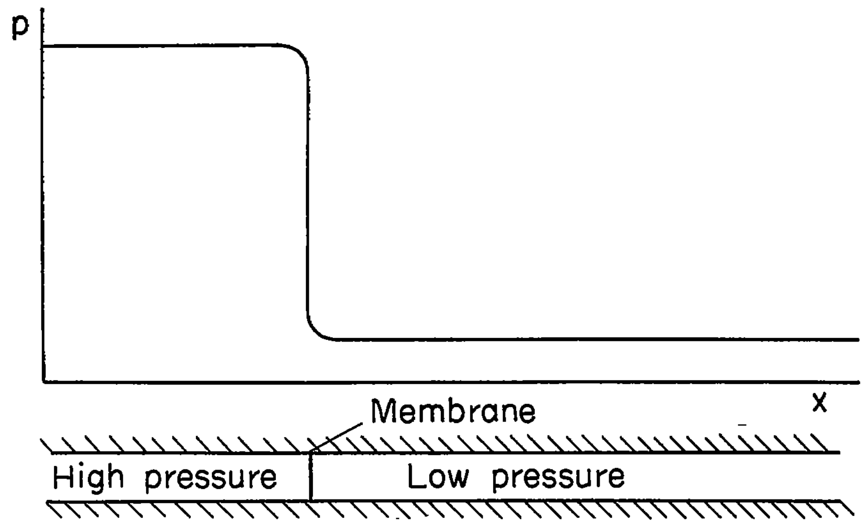

Figure 8.- Initial state in the shock tube.

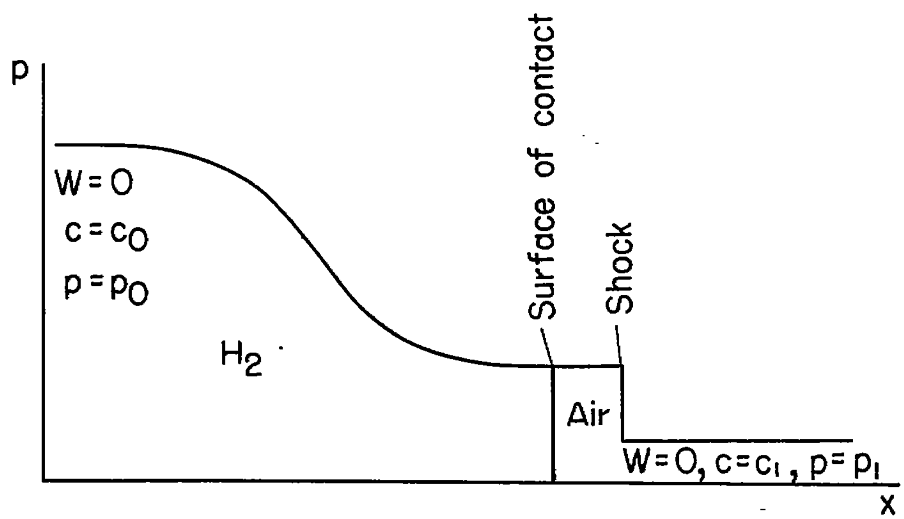

Figure 9.- Explosion process in the tube.

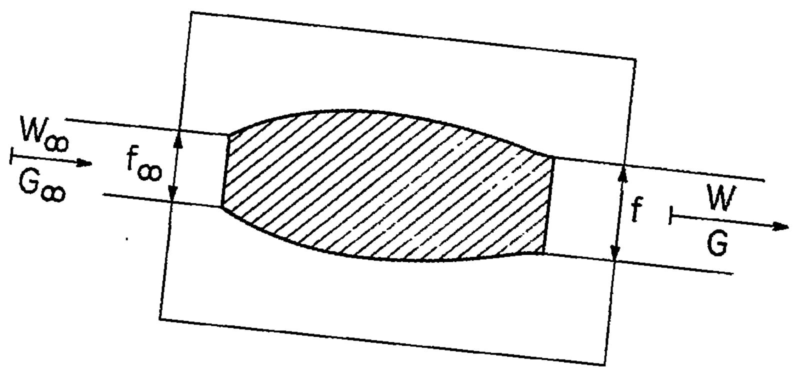

Figure 10.- Thrust body.